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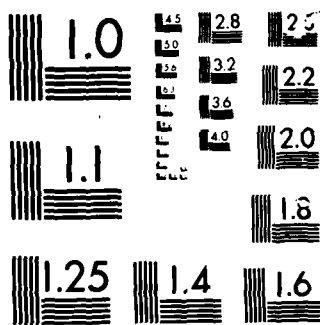
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ON THE EXISTENCE OF PERIODIC SOLUTIONS
FOR A CLASS OF SYMMETRIC HAMILTONIAN
SYSTEMS

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ON THE EXISTENCE OF PERIODIC SOLUTIONS FOR A CLASS
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ABSTRACT

The main result in this paper is:

Theorem: If $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ and satisfies

(H₁) $H^{-1}(1)$ bounds a starshaped neighborhood of 0 in \mathbb{R}^{2n} ,

(H₂) $z \cdot H_z \neq 0$ for all $z \in H^{-1}(1)$,

(H₃) $H(p, q) = H(-p, q)$ for all $p, q \in \mathbb{R}^n$, then there is a $T > 0$ such that the Hamiltonian system

$$(HS) \quad \dot{z} = H_z(z), \quad z = \begin{pmatrix} 0 & -id \\ id & 0 \end{pmatrix}$$

possesses a T periodic solution $(p(t), q(t)) \in H^{-1}(1)$ with p odd about 0 and $T/2$ and q even about 0 and $T/2$.

The proof involves a new existence mechanism which should be useful in other situations.

AMS (MOS) Subject Classifications: 34C25, 58E05, 58F22, 70H05, 70H25, 70K99

Key Words: periodic solution, Hamiltonian system, minimax methods

Work Unit Number 1 (Applied Analysis)

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SIGNIFICANCE AND EXPLANATION

Hamiltonian systems are used to model the motion of discrete mechanical systems. This paper establishes the existence of periodic solutions for a class of such systems. The method developed to prove existence should be useful for other such problems.

*Keywords: minimum techniques;
ordinary differential equations; variational problems.*

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ON THE EXISTENCE OF PERIODIC SOLUTIONS FOR A CLASS OF SYMMETRIC HAMILTONIAN SYSTEMS

Paul H. Rabinowitz

§1. Introduction

Consider the Hamiltonian system of ordinary differential equations:

$$(HS) \quad \dot{z} = JH_z(z), \quad J = \begin{pmatrix} 0 & -id \\ id & 0 \end{pmatrix}.$$

Here $H : \mathbb{R}^{2n} \rightarrow \mathbb{R}$, $z = (p, q)$ with $p, q \in \mathbb{R}^n$, and id denotes the $n \times n$ identity matrix. Several papers have investigated what conditions on H lead to the existence of periodic solutions of (HS) having prescribed energy, i.e. $H(z)$ is a given constant. See e.g. [1-11]. (Other studies such as [12] treat the multiplicity of periodic solutions of (HS) of prescribed energy.) In particular, it was shown in [4] that

Theorem 1.1: If $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ and satisfies

(H₁) $H^{-1}(1)$ is the boundary of a starshaped neighborhood of 0 in \mathbb{R}^{2n} ,

and

(H₂) $z \cdot H_z(z) \neq 0$ on $H^{-1}(1)$,

then (HS) possesses a periodic solution on $H^{-1}(1)$.

In Theorem 1.1, "starshaped" means $H^{-1}(1)$ is homeomorphic to S^{2n-1} by a radial projection map.

Our goal in this paper is to show that if H satisfies an additional symmetry condition, (HS) possesses a periodic solution having additional properties:

Theorem 1.2: If $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ and satisfies (H₁)-(H₂) and

(H₃) $H(p, q) = H(-p, q)$ for all $p, q \in \mathbb{R}^n$,

then there exists a $T > 0$ and a T periodic solution $(p(t), q(t))$ of (HS) on $H^{-1}(1)$ such that p is odd and q is even about $t = 0$ and $\frac{T}{2}$.

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Periodic solutions of this type were studied by SEIFERT [1], RUIZ [2], WEINSTEIN [3], GLUCK-ZILLER [7], HAYASHI [8], and BENCI [9] for C^2 or smoother H 's satisfying (H_3) and having the form $H(p,q) = K(p,q) + V(q)$ with K and V suitably restricted. Different symmetries for (HS) have been treated by van GROESEN [10] and GIRARDI [11]. In [10] it was shown that if $H \in C^2$, satisfies (H_2) , $H^{-1}(1)$ bounds a convex region and $H(p,q) = H(-p,q) = H(p,-q)$ for all $p,q \in \mathbb{R}^n$, then the conclusions of Theorem 1.2 hold. The convexity assumption on $H^{-1}(1)$ plays a strong role in the existence argument here. In [11] on the other hand, $(H_1)-(H_2)$ and $H(z) = H(-z)$ are assumed and it is proved that there is a $\tau > 0$ such that (HS) possesses a 2τ periodic solution on $H^{-1}(1)$ for which $z(t + \tau) = -z(t)$ for all $t \in \mathbb{R}$.

Both [4] and [11] rely on minimax arguments and topological index theories to exploit an S^1 symmetry associated with a variational formulation of (HS). Topological index theories are often useful in obtaining multiple critical points of a symmetric functional and indeed such multiplicity results were the main goal of [10-11] and enabled them to obtain analogues of a theorem of EKELAND and LASRY [12]. For the problem treated here however, we will work directly in the space of T periodic functions p,q for which p is odd about $t = 0$ and $T/2$ and q is even about 0 and $T/2$. The symmetries used earlier in [4,10,11] are not longer present if one works in this space and therefore another existence mechanism to treat (HS) is required. Indeed developing such a new mechanism is one of our main goals here. In a future note, we will show how this method can also be applied to treat the sort of situation studied in [1-3,5, etc.].

In §2, the solution of (HS) will be reduced to finding a critical point of an associated variational problem. Existence of a critical point when $H \in C^2$ is carried out in §3. Lastly §4 contains the C^1 case as well as the proof of a crucial intersection theorem used in §3. For some of the technicalities of §2-4, we have benefited from unpublished work of V. BENCI and the author.

§2. Formulation of the Variational Problem

In this section the solution of (HS) will be reduced to finding critical points of a variational problem. For technical convenience we assume for now that $H \in C^2(\mathbb{R}^{2n}, \mathbb{R})$. The more general case of $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ will be treated in §4.

The variational formulation of (HS) will take place in an " L^2 " type of setting and therefore the behavior of H outside of $H^{-1}(1)$ is important. Thus as in [4] we define a new Hamiltonian $\bar{H}(z)$ which coincides with H on $H^{-1}(1)$ and grows at a controlled rate as $|z| \rightarrow \infty$. Since $H^{-1}(1)$ bounds a starshaped region, for all $z \in \mathbb{R}^{2n} \setminus \{0\}$, there is a unique $\alpha(z) > 0$ and $w(z) \in H^{-1}(1)$ such that $z = \alpha(z)w(z)$. In fact, w depends only on $z|z|^{-1}$ and $\alpha(z) = |z||w(z)|^{-1}$. Define $\bar{H}(0) = 0$ and for $z \neq 0$, $\bar{H}(z) = \alpha(z)^2$. It is easy to check that $\bar{H} \in C^2(\mathbb{R}^{2n} \setminus \{0\}, \mathbb{R})$, \bar{H}_{zz} is uniformly bounded, and \bar{H} is homogeneous of degree two. Moreover $\bar{H}^{-1}(1) = H^{-1}(1)$ and $\bar{H}(p, q)$ is even in p . For future reference, note the following estimates for \bar{H} . Let

$$(2.1) \quad m \equiv \min_{|z|=1} \bar{H}(z); \quad M = \max_{|z|=1} \bar{H}(z).$$

Then by the homogeneity of \bar{H} , for all $z \in \mathbb{R}^{2n}$,

$$(2.2) \quad m|z|^2 < \bar{H}(z) < M|z|^2.$$

Lemma 2.3: Suppose there exists $\lambda > 0$ and a 2π periodic solution $\zeta(\tau) = (\varphi(\tau), \psi(\tau))$

$$(2.4) \quad \dot{\zeta} = \lambda J \bar{H}_z(\zeta)$$

with $\zeta(\tau) \in \bar{H}^{-1}(1)$, φ odd about 0 and π and ψ even about 0 and π . Then there exists a $T > 0$ and a T periodic solution of (HS) of the type stated in Theorem 1.2.

Proof: Set $z(t) = \zeta(r(t))$ where $r \in C^1$ is free for the moment. Then z is a solution of (HS) if

$$(2.5) \quad \dot{z} = \lambda J \bar{H}_z(\zeta(r(t))) \dot{r} = J H_z(\zeta(r(t))).$$

Since $H^{-1}(1) = \bar{H}^{-1}(1)$ and $H_z, \bar{H}_z \neq 0$ on this set, there is a function

$\beta \in C^1(H^{-1}(1), \mathbb{R} \setminus \{0\})$ such that $\bar{H}_z(z) = \beta(z) \bar{H}_z(z)$ for $z \in H^{-1}(1)$. Therefore 2.5 shows

$$(2.6) \quad \dot{r} = \lambda^{-1} \beta(\zeta(r(t))).$$

Further setting $r(0) = 0$ and noting that $\beta \neq 0$, we can assume $\beta > 0$ and r is a

strictly increasing function of t . Let $T = 2\pi^{-1}(\pi)$. Then the properties of ζ imply if $Z(t) = (P(t), Q(t))$, $P(0) = 0 = P(T/2)$ and $Q'(0) = 0 = Q'(T/2)$. Extending P as an odd function p about 0 and $T/2$ and Q as an even function q about 0 and $T/2$, it follows that the resulting function $z = (p, q)$ is a T periodic solution of (HS) on $H^{-1}(1)$ of the desired type.

Thus Lemma 2.3 reduces the proof of Theorem 1.2 to finding $\lambda > 0$ and a 2π periodic solution ζ of (2.4). We will convert this question to that of solving a variational problem. First an appropriate function space must be introduced. Let $X = \{z = (p, q) \in W^{1/2, 2}(S^1, \mathbb{R}^{2n}) \mid p \text{ is odd about } 0 \text{ and } \pi, q \text{ is even about } 0 \text{ and } \pi\}$. Here $W^{1/2, 2}(S^1, \mathbb{R}^{2n})$ is the set of $2n$ tuples of 2π periodic functions

$$z = \sum_{j \in \mathbb{Z}} a_j e^{ij t}$$

such that

$$\sum_{j \in \mathbb{Z}} (1 + |j|) |a_j|^2 < \infty.$$

For smooth $z \in X$, let

$$(2.7) \quad A(z) = \int_0^{2\pi} p \cdot \dot{q} \, dt.$$

Then

$$|A(z)| \leq \text{const.} \|z\|_{W^{1/2, 2}}^2,$$

i.e. A is a continuous quadratic form on this (dense) subspace of X . Therefore A extends continuously to all of X . This extension will still be denoted by $A(z)$.

Let e_1, \dots, e_{2n} denote the usual basis in \mathbb{R}^{2n} , i.e. $e_1 = (1, 0, 0, \dots)$, etc. and set

$$X_0 \equiv \text{span}\{e_k \mid n+1 \leq k \leq 2n\},$$

$$X^+ \equiv \text{span}\{(\sin jt)e_k - (\cos jt)e_{k+n} \mid 1 \leq j < \infty, 1 \leq k \leq n\},$$

$$X^- \equiv \text{span}\{(\sin jt)e_k + (\cos jt)e_{k+n} \mid 1 \leq j < \infty, 1 \leq k \leq n\}.$$

These spaces are mutually orthogonal in $L^2(S^1, \mathbb{R}^{2n})$. Moreover $X = X^0 \oplus X^+ \oplus X^-$ and if $z = z^0 + z^+ + z^- \in X$,

$$(2.8) \quad \|z\|^2 \equiv |z^0|^2 + A(z^+) - A(z^-)$$

defines a norm on X which is equivalent to the $W^{1/2,2}(S^1, \mathbb{R}^{2n})$ norm. (See e.g. [4].)

Setting

$$(2.9) \quad \Psi(z) \equiv \frac{1}{2\pi} \int_0^{2\pi} \bar{H}(z) dt,$$

the upper bound for \bar{H} in (2.2) implies Ψ is well defined on X .

Proposition 2.10: With \bar{H} as above,

(i) $\Psi \in C^{1, \text{Lip}}(X, \mathbb{R})$,

(ii) Ψ' is compact.

Proof: (i) Since $\bar{H} \in C^2(\mathbb{R}^{2n} \setminus \{0\}, \mathbb{R})$ and \bar{H}_{zz} is uniformly bounded, there is a constant $M_1 > 0$ such that

$$(2.11) \quad |\bar{H}(z + \zeta) - \bar{H}(z) - \bar{H}_z(z) \cdot \zeta| \leq M_1 |\zeta|^2$$

for all $z, \zeta \in \mathbb{R}^{2n}$. Therefore for $z, \zeta \in X$, (2.11) and the continuous embedding of X in $L^2(S^1, \mathbb{R}^{2n})$ imply that

$$(2.12) \quad |\Psi(z + \zeta) - \Psi(z) - \frac{1}{2\pi} \int_0^{2\pi} \bar{H}_z(z) \cdot \zeta dt| \leq M_1 \|\zeta\|_{L^2}^2 \leq M_2 \|\zeta\|^2.$$

Thus (2.12) shows that $\Psi \in C^1(X, \mathbb{R})$ and

$$(2.13) \quad \Psi'(z)\zeta = \frac{1}{2\pi} \int_0^{2\pi} \bar{H}_z(z) \cdot \zeta dt.$$

To see that Ψ' is Lipschitz continuous, observe that

$$(2.14) \quad \|\Psi'(z+w) - \Psi'(z)\|_* = \sup_{\zeta \in X, \|\zeta\| \leq 1} \left| \frac{1}{2\pi} \int_0^{2\pi} (\bar{H}_z(z+w) - \bar{H}_z(z)) \cdot \zeta dt \right|.$$

Since \bar{H}_{zz} is uniformly bounded on $\mathbb{R}^{2n} \setminus \{0\}$, there is a constant $M_3 > 0$ such that

$$(2.15) \quad |\bar{H}_z(z+w) - \bar{H}_z(z)| < M_3 |w|$$

for all $z, w \in \mathbb{R}^{2n}$. Hence (2.14)-(2.15) imply

$$(2.16) \quad \|\Psi'(z+w) - \Psi'(z)\|_{X^*} < M_3 \sup_{\zeta \in X, \|\zeta\| \leq 1} \frac{1}{2\pi} \int_0^{2\pi} |w| |\zeta| dt < M_4 \|w\|$$

for all $z, w \in X$.

(ii) Let (z_j) be a bounded sequence in X . Since X is compactly embedded in $L^r(S^1, \mathbb{R}^{2n})$ for all $r \in [1, \infty)$, (see e.g. the argument for an analogous situation in [13]), along a subsequence, z_j converges in $L^2(S^1, \mathbb{R}^{2n})$ to $z \in X$. Hence by (2.16),

$$(2.17) \quad \|\Psi'(z_j) - \Psi'(z)\|_{X^*} < M_4 \|z_j - z\|_{L^2} \rightarrow 0$$

and Ψ' is compact.

Let $M \equiv \Psi^{-1}(1)$.

Proposition 2.18: (i) M is a $C^{1, \text{Lip}}$ manifold in X .

(ii) M bounds a starshaped neighborhood of 0 in X .

(iii) M is bounded in $L^2(S^1, \mathbb{R}^{2n})$.

Proof: For $z \in X \setminus \{0\}$, by the homogeneity of \bar{H} ,

$$(2.19) \quad \Psi'(z)z = \frac{1}{2\pi} \int_0^{2\pi} \bar{H}_z(z) \cdot z \, dt = 2\Psi(z) > 0.$$

Hence M is a manifold and (i) of Proposition 2.10 shows it is $C^{1, \text{Lip}}$. Moreover M is the boundary of $\Psi^{-1}(-\infty, 1)$, an open set. The homogeneity of \bar{H} shows that any ray through the origin in X meets M exactly once. Hence M bounds a starshaped region.

Lastly by (2.2), if $z \in M$,

$$(2.20) \quad \frac{m}{2\pi} \|z\|_{L^2}^2 < \Psi(z) = 1,$$

i.e. M is bounded in $L^2(S^1, \mathbb{R}^{2n})$.

We will find a solution of the desired type as a critical point of $A|_M$. This functional is said to satisfy the $(PS)^+$ condition if for any $c > 0$, whenever (z_j) is a sequence in M such that

$$(2.21) \quad A(z_j) \rightarrow c$$

and

$$(2.22) \quad A|_M'(z_j) \equiv A'(z_j) - \lambda(z_j)\Psi'(z_j) \rightarrow 0 \text{ in } X^*$$

where

$$\lambda(z) = (A'(z), \Psi'(z))_{X^*, X}^{-2},$$

then (z_j) possesses a convergent subsequence. Thus the $(PS)^+$ condition is a kind of compactness condition. It is important that $A|_M$ satisfy this condition in order to construct the "deformation mapping" used in §3.

Proposition 2.23: $A|_M$ satisfies the $(PS)^+$ condition.

Proof: Let $(z_j) \subset M$ and satisfy (2.21)-(2.22). Writing $z_j = z_j^+ + z_j^- + z_j^0 \in X^+ \oplus X^- \oplus X^0$, (2.22) and the homogeneity of A show

$$(2.24) \quad \|z_j^+\|^2 = A'(z_j)(z_j^+) < |\lambda(z_j)| \int_0^{2\pi} \bar{H}_z(z_j) \cdot z_j^+ dt + \varepsilon_j \|z_j^+\|$$

where $\varepsilon_j \rightarrow 0$ as $j \rightarrow \infty$. Since (z_j) is bounded in $L^2(S^1, \mathbb{R}^{2n})$ as is $(\bar{H}_z(z_j))$ via (2.15), by (2.24) and (2.8),

$$(2.25) \quad \|z_j^+\|^2 < a_1(|\lambda(z_j)| + 1).$$

Now by the homogeneity of A and \bar{H} ,

$$(2.26) \quad 2|A(z_j) - \lambda(z_j)| = |A'(z_j)z_j - \lambda(z_j)\Psi'(z_j)z_j| < \varepsilon_j \|z_j\|.$$

Combining (2.25) and (2.26) gives

$$(2.27) \quad |A(z_j) - \lambda(z_j)| < a_2(|\lambda(z_j)| + 1)^{1/2}.$$

Recalling that $A(z_j) \rightarrow c$, (2.27) shows $\lambda(z_j)$ is a bounded sequence and (2.25)

implies (z_j) is bounded in X . Consequently (2.26) yields $\lambda(z_j) \rightarrow c > 0$. Let l denote the duality map from X^* to X . Then

$$(2.28) \quad L(\Lambda'(z_j) - \lambda(z_j)\Psi'(z_j)) = z_j^+ - z_j^- - \lambda(z_j)L\Psi'(z_j) + 0.$$

Thus the boundedness of $(\lambda(z_j))$ and (z_j) , (ii) of Proposition 2.10, and (2.28) show z_j^+ , z_j^- , and - since X^0 is finite dimensional - z_j^0 converge along a subsequence.

§3. Existence of a Solution

In this section, the existence of a critical point of $A|_M$ will be established. Standard arguments then lead to a solution of (2.4) and hence (by Lemma 2.3) of (HS) of the desired type. A minimax argument will be used to get a critical point of $A|_M$. An important role in any minimax argument is played by the so-called deformation mapping. The following proposition lists its properties in our setting. For $c > 0$, let

$$A_c = \{z \in M | A(z) < c\} \text{ and}$$

$$K_c = \{z \in M | A(z) = c \text{ and } A'_M(z) = 0\}.$$

Proposition 3.1: Let $c, \bar{c} > 0$. Then there exists an $\varepsilon \in (0, \bar{c})$ and $\eta \in C([0, 1] \times X, X)$ such that

1° $\eta(s, \cdot)$ is a homeomorphism of X onto X for each $s \in [0, 1]$.

2° $\eta(1, z) = z$ if $A(z) \notin [c - \bar{c}, c + \bar{c}]$ and if $|\Psi(z) - 1| > \frac{1}{2}$.

3° $|\eta(1, z) - z| < 1$.

4° $\eta(s, M) = M$ for each $s \in [0, 1]$.

5° If P^+, P^- denote respectively the orthogonal projectors of X onto X^+, X^- , then

$$P^+ \eta(s, z) = e^{i\theta(s, z)} z^+ + K^\pm(s, z)$$

where $\theta \in C([0, 1] \times X, \mathbb{R}^+)$ and K^\pm is compact.

6° If $K_c = \emptyset$, $\eta(1, A_{c+\varepsilon}) \subset A_{c-\varepsilon}$.

Proof: Most of the assertions are standard. In particular 1° and 6° as well as the precise definition of ω below can be found in [14]. (It is in proving 6° that the $(PS)^+$ condition is used.) Therefore we will only verify 2°-5°.

The function η satisfies an ordinary differential equation of the form

$$(3.2) \quad \frac{d\eta}{ds} = -\omega(\eta) [A'(\eta) - \lambda(\eta) \Psi'(\eta)] \quad \eta(0, z) = z$$

for $z \in X$. The function $\omega \in C(X, \mathbb{R})$ is Lipschitz continuous and is chosen so that $0 \leq \omega(z) < 1$, the right-hand side of (3.2) in norm does not exceed 1, $\omega(z) = 0$ if $A(z) \notin [c - \bar{c}, c + \bar{c}]$ or if $|\Psi(z) - 1| > \frac{1}{2}$, and $\omega(z) \neq 0$ if $z \in M$ and $A(z)$ is near c . This implies that (3.2) has a solution $\eta(s, z) \in C(\mathbb{R} \times X, X)$ satisfying 2°-3°. The form of the right-hand side of (3.2) shows $\Psi'(\eta(s, z)) \frac{d\eta}{ds} \equiv 0$ and therefore

$\Psi(\eta(s,z)) \equiv \Psi(\eta(0,z)) = \Psi(z)$. In particular if $z \in M$, so is $\eta(s,z)$ and 4° holds.

Lastly to prove 5° , note that $LA'(z) = z^+ - z^-$. Therefore $P^\pm \eta \equiv \eta^\pm$ satisfies

$$(3.3) \quad \frac{d\eta^\pm}{ds} \pm \omega(\eta)\eta^\pm = \omega(\eta)\lambda(\eta)P^\pm L\Psi'(\eta).$$

Treating η as being known, (3.3) shows η^\pm satisfies an inhomogeneous linear equation whose solution is

$$(3.4) \quad \eta^\pm(s,z) = [\exp(\mp \int_0^s \omega(\eta(r,z))dr)]z^\pm + K^\pm(s,z)$$

where

$$K^\pm(s,z) = \int_0^s [\exp(\mp \int_s^\tau \omega(\eta(r,z))dr)]S^\pm(\eta(\tau,z))d\tau$$

and

$$S^\pm(y) = \omega(y)\lambda(y)P^\pm L\Psi'(y).$$

Thus η is of the form asserted in 5° .

It remains only to show that K^\pm is compact. Note first that $S^\pm : X \rightarrow X$ is compact. For convenience we drop the superscripts \pm for S and K . Indeed if (y_j) is bounded in X and $\Psi(y_j) \notin (\frac{1}{2}, \frac{3}{2})$ along some subsequence, then $\omega(y_j) = 0$ and $S(y_j) = 0$. Thus we can assume $\Psi(y_j) \in (\frac{1}{2}, \frac{3}{2})$ for all $j \in \mathbb{N}$. Since $\Psi(0) = 0$, this implies (y_j) is bounded away from 0. Therefore

$$\|\Psi'(y_j)\|_{X^*} > \Psi'(y_j) \frac{y_j}{\|y_j\|} = \frac{2\Psi(y_j)}{\|y_j\|}$$

is bounded away from 0. It follows that $(\lambda(y_j))$ is a bounded sequence and therefore by (ii) of Proposition 2.10, $S(y_j)$ has a convergent subsequence.

To get the compactness of K , we use a variant of an argument of BENCI [15]. Let $B \subset X$ be bounded. Without loss of generality, $B = B_R$, a ball of radius R about 0. By 3° of Proposition 3.1, $\eta([0,1] \times B_R) \subset B_{R+1}$. Therefore $S(\eta([0,1] \times B_R)) \subset \overline{S(B_{R+1})}$ which is compact. If

$$Y \equiv \{aw \mid a \in [0,1], w \in \overline{S(B_{R+1})}\},$$

then Y is compact as is \hat{Y} , its closed convex hull. Recalling that $\omega(\xi) \in [0,1]$, it follows that for each $\tau \in [0,s]$ and $z \in B_R$,

$$Z \equiv \{\exp(\int_s^\tau \omega(\eta(r,z))dr)\} S(\eta(\tau,z)) \in Y.$$

Hence for $s \in [0,1]$,

$$\int_0^s Z d\tau \in \hat{Y},$$

K is compact, and the proof is complete.

Now define $M^+ \equiv X^+ \cap M$,

$$W = X^0 \oplus X^- \oplus \text{span}\{\varphi\}$$

where $\varphi = (\sin t)e_1 - (\cos t)e_{n+1} \in X^+$. Set $M^- = W \cap M$. Define

$$(3.6) \quad \underline{\alpha} \equiv \inf_{z \in M^+} A(z)$$

and

$$(3.7) \quad \bar{\alpha} \equiv \sup_{z \in M^-} A(z).$$

Proposition 3.8: $0 < \underline{\alpha} < \bar{\alpha} < \infty$.

Proof: If $z \in X^+$, $A(z) = |z|^2$. If $\underline{\alpha} = 0$, $0 \in M^+$, $= M^+$. Since by (ii) of Proposition 2.18, M bounds a neighborhood of 0 in X , this is impossible and $\underline{\alpha} > 0$. Next observe that

$$\underline{\alpha} < \inf_{\text{span}\{w\} \cap M} A(z) < \sup_{M^-} A(z) = \bar{\alpha}.$$

Finally note that if $z \in M^-$, $z = r(z)\varphi + z^0 + z^-$. Therefore since $A(\varphi) = \pi$,

$$(3.9) \quad A(z) < r^2(z)\pi.$$

Since $z \in M$, by (2.2),

$$(3.10) \quad \frac{1}{2\pi} \int_0^{2\pi} m|z|^2 dt < 1 = \Psi(z) .$$

Hence (3.10) and the orthogonality of x^0, x^\pm in L^2 imply

$$(3.11) \quad \frac{2\pi}{m} > r(z)^2 \int_0^{2\pi} |\varphi|^2 dt = 2\pi r(z)^2 .$$

Therefore (3.7) and (3.11) show

$$(3.12) \quad \bar{a} < \pi r(z)^2 < \frac{\pi}{m} .$$

Our goal is to obtain a critical value c of $A|_M$ via a minimax argument. The class of maps which will be used to define c can now be introduced. Let Γ denote the set of $h \in C(X, X)$ satisfying the following three conditions:

$$(\Gamma_1) \quad h(z) = z \text{ if } A(z) \notin [0, \bar{a} + 1] \text{ or if } |\Psi(z) - 1| > \frac{1}{2} .$$

$$(\Gamma_2) \quad P^-h(z) = e^{\varphi(z)} z^- + Q(z) \text{ where } \varphi \in C(X, \mathbb{R}^+), 0 < \varphi < \gamma, \gamma \text{ depending on } h, \text{ and } Q \text{ is compact.}$$

$$(\Gamma_3) \quad h : M \rightarrow M .$$

Remark 3.13: Observe that Γ is closed under composition. Moreover 1⁰⁻⁵⁰ of Proposition 3.1 imply $\eta(1, \cdot) \in \Gamma$ provided that $0 < c - \bar{e} < c + \bar{e} < \bar{a} + 1$. This inequality holds in particular if $c \in (0, \bar{a}]$ and \bar{e} is chosen appropriately.

The mappings in Γ satisfy an important intersection property.

Proposition 3.14: If $h \in \Gamma$, then $h(M^-) \cap M^+ \neq \emptyset$.

This proposition will be proved in §4. Assuming it for now, define

$$(3.15) \quad c = \inf_{h \in \Gamma} \sup_{z \in M^-} A(h(z)) .$$

Proposition 3.16: $\underline{a} < c < \bar{a}$ and c is a critical value of $A|_M$.

Proof: If $h \in \Gamma$, by Proposition 3.14,

$$(3.17) \quad \sup_{z \in M^-} A(h(z)) > \sup_{\zeta \in h(M^-) \cap M^+} A(\zeta) > \inf_{\zeta \in M^+} A(\zeta) = \underline{a} .$$

Since (3.17) holds for all $h \in \Gamma$, $c > \underline{a}$. On the other hand $h(z) \equiv z \in \Gamma$. Consequently

$$(3.18) \quad c \leq \sup_{z \in M^-} \Lambda(z) = \bar{a}.$$

To prove that c is a critical value of $\Lambda|_M$, suppose on the contrary that $K_c = \emptyset$.

Then if $\bar{\epsilon} < \min(\frac{1}{2}\bar{a}, 1)$, Remark 3.13 shows $\eta(1, \cdot)$ as determined from Proposition 3.1 belongs to Γ . Choose $h \in \Gamma$ such that

$$(3.19) \quad \sup_{z \in M^-} \Lambda(h(z)) < c + \epsilon$$

where ϵ is obtained from Proposition 3.1. Since $\eta(1, h) \in \Gamma$, by 6° of Proposition 3.1,

(3.15), and (3.19),

$$(3.20) \quad c \leq \sup_{z \in M^-} \Lambda(\eta(1, h(z))) < c - \epsilon$$

a contradiction and the proposition is proved.

Completion of proof of Theorem 1.2 for $H \in C^2$: By Proposition 3.16, there is a $z \in M$ such that $\Lambda(z) = c$ and

$$(3.21) \quad (\Lambda'(z) - \lambda(z)\nabla'(z))(\zeta) = 0$$

for all $\zeta \in X$. This implies z is a classical solution of (2.4). Indeed $z \in X$

implies $\bar{H}_z(z) \in L^2(S^1, \mathbb{R}^{2n})$. Moreover by (H_3) , $\bar{H}_p(p(t), q(t))$ is odd about 0 and π and $\bar{H}_q(p(t), q(t))$ is even about 0 and π . Taking $\zeta \equiv 1$ in (3.21) yields

$$[\bar{H}_z(z(t))] = 0 \text{ where}$$

$$[w] = \frac{1}{2\pi} \int_0^{2\pi} w(t) dt.$$

Since $[\bar{H}_p(p(t), q(t))] = 0$, Fourier expansion shows the equations

$$(3.22) \quad \begin{aligned} (i) \quad \frac{dp}{dt} &= -\lambda(z)\bar{H}_q(p(t), q(t)) \\ (ii) \quad \frac{dq}{dt} &= \lambda(z)\bar{H}_p(p(t), q(t)) \end{aligned}$$

have a unique solution $Z = (P, Q) \in X \cap W^{1,2}(S^1, \mathbb{R}^{2n})$ with $[Q] = [q]$. For smooth

$\zeta = (\varphi, \psi) \in X$, taking the inner product of (3.22) (i) with φ and (ii) with ψ gives

$$(3.23) \quad (A'(Z) - \lambda(z) \nabla'(z))(\zeta) = 0.$$

Comparing (3.21) and (3.23) shows

$$(3.24) \quad \int_0^{2\pi} [(p - P) \cdot \dot{\psi} - (q - Q) \cdot \dot{\phi}] dt = 0$$

for all smooth $(\psi, \phi) \in X$ where $[p - P] = 0 = [q - Q]$. Hence $z = Z \in W^{1,2}(S^1, \mathbb{R}^{2n}) \subset C(S^1, \mathbb{R}^{2n})$. Thus (3.22) shows $z \in C^1(S^1, \mathbb{R}^{2n})$ and is a classical solution of (2.4). But (2.4) is a Hamiltonian system so $\bar{H}(z(t))$ is independent of t . Consequently

$$(3.25) \quad 1 = \nabla(z) = \frac{1}{2\pi} \int_0^{2\pi} \bar{H}(z(t)) dt = \bar{H}(z(t))$$

so $z(t) \in \bar{H}^{-1} = H^{-1}(1)$. Finally Lemma 2.3 gives a solution of the desired type of (HS).

§4. The Intersection Theorem and the General Case of Theorem 1.2

In this section we will prove the intersection theorem: Proposition 3.14, and obtain the C^1 case of Theorem 1.2. For the former result, the following technical result is required:

Proposition 4.1: Let V be a k dimensional subspace of \mathbb{R}^n and V^\perp its orthogonal complement. Suppose $h \in C(\mathbb{R}^n, \mathbb{R}^n)$ and satisfies

$$(h_1) \quad h = \text{id} \text{ on } V^\perp$$

and

$$(h_2) \quad \text{there is an } R > 0 \text{ such that } h = \text{id} \text{ on } \mathbb{R}^n \setminus B_R.$$

Let $\psi \in C^1(\mathbb{R}^n, \mathbb{R})$ and satisfy

$$(\psi_1) \quad \psi(0) = 0 \text{ and } x \cdot \psi'(x) > 0 \text{ for } x \neq 0$$

and

$$(\psi_2) \quad \text{there is a } \rho \in (0, R) \text{ such that } \psi^{-1}(\rho) \subset B_R.$$

Let $v \in V \cap \partial B_\rho$ and set $Y = \text{span}\{v\} \oplus V^\perp$. Then there is a $\xi \in Y$ such that

$$(4.2) \quad \psi(\xi) = \rho \text{ and } h(\xi) \in V.$$

Proof: Let $Q \equiv \{rv \mid 0 < r < R\} \oplus (B_R \cap V^\perp)$ so $Q \subset Y$. We will find $\xi \in Q$. Let P and P^\perp denote the orthogonal projectors of \mathbb{R}^n onto V and V^\perp . Solving (4.2) for $\xi \in Y$ is equivalent to finding $\xi \in Y$ such that

$$(4.3) \quad \begin{aligned} (i) \quad & \psi(\xi) = \rho \\ (ii) \quad & P^\perp h(\xi) = 0. \end{aligned}$$

For $y \in Y$, set

$$\Phi(y) = (\psi(y), P^\perp h(y)).$$

Identifying $\mathbb{R} \times Y$ with $\mathbb{R} \times \mathbb{R}^{n-k}$ and Q with a subset thereof, Φ can be considered to be a continuous map of $\mathbb{R} \times \mathbb{R}^{n-k}$ into itself. Any zero of Φ is a solution of (4.3). Consider $d(\Phi, Q, (\rho, 0))$, the Brouwer degree of Φ with respect to the bounded open set Q and the point $(\rho, 0)$. We will show this degree equals 1 and therefore (4.3) has a solution in Q . In order for the degree to be defined, it is necessary that $\Phi \neq (\rho, 0)$ on ∂Q . Writing $y \in Q$ as $(r, w) \in \mathbb{R} \times \mathbb{R}^{n-k}$, if $r = 0$, by (h_1) , $h = \text{id}$

so by (ψ_1) , $\phi(y) = (\psi(w), w) \neq (\rho, 0)$. If $r = R$, (h_2) implies $h = \text{id}$ and $\phi(y) = (\psi(Rv + w), w) \neq (\rho, 0)$ since $\psi(Rv) > \rho$ via $(\psi_1) - (\psi_2)$. Finally if $|w| = R$, (h_2) implies $h = \text{id}$ and $\phi(y) = (\psi(rv + w), w) \neq (\rho, 0)$. Therefore $d(\phi, Q, (\rho, 0))$ is defined. We claim

$$(4.4) \quad d(\phi, Q, (\rho, 0)) = d(\text{id}, Q, (\rho, 0)) .$$

Since $(\rho, 0) \in Q$ via (ψ_2) ,

$$d(\text{id}, Q, (\rho, 0)) = 1$$

and the proof is complete. To verify (4.4), consider the homotopy

$$\phi_\theta(y) = (\theta r + (1 - \theta)\psi(y), P^1 h(y))$$

for $y \in \bar{Q}$ and $\theta \in [0, 1]$. Arguing as for ϕ , if $r = 0$,

$$\phi_\theta(y) = ((1 - \theta)\psi(w), w) \neq (\rho, 0); \text{ if } r = R, \phi_\theta(y) = (\theta R + (1 - \theta)\psi(Rv + w), w) \neq (\rho, 0)$$

since $R, \psi(Rv) > \rho$; if $|w| = R$, $\phi_\theta(y) = (\theta r + (1 - \theta)\psi(rv + w), w) \neq (\rho, 0)$. Since

$\phi_0(y) = \phi(y)$ and $\phi_1(y) = y$ on ∂Q , the homotopy invariance property of Brouwer degree yields (4.4) and the proof is complete.

Remarks 4.5: (i) The same hypotheses and an appropriately modified ϕ also yield an $y \in Y$ such that $h(y) \in V \cap \psi^{-1}(\rho)$. This fact can be used to obtain a critical value of $\lambda|_M$ as a maximin rather than a minimax. (ii) An examination of the proof of Proposition 4.1 shows we need merely take $\psi \in C(\mathbb{R}^n, \mathbb{R})$ and weaken (ψ_1) to $\psi(0) = 0$ and $\psi(Rv) > \rho$ for some $v \in V \cap \partial B_1$. Also at the expense of redefining B_R, V and V^\perp can be any complementary subspaces of \mathbb{R}^n .

Now we can give the:

Proof of Proposition 3.14: Let X_1^\pm denote the subspaces of X^\pm defined by restricting j to $1 \leq j \leq i$ in the definition of X^\pm . Let $X_1 = X^0 \oplus X_1^+ \oplus X_1^-$ and let P_1 denote the projector of X onto X_1 . Define $h_1(z) \equiv P_1 h(z)$. By (2.2),

$$\forall(z) > \frac{m}{2\pi} |z|_2^2 .$$

Hence $\forall(z) \rightarrow \infty$ as $|z| \rightarrow \infty$ uniformly for $z \in X_1$. In particular there is an $R_1 > 0$ such that $\forall(z) > 2$ if $z \in X_1$ and $|z| > R_1$. Therefore if $h \in \Gamma$, by (Γ_1) ,

$h(z) = z = h_1(z)$ if $z \in X_1$ and $|z| > R_1$. Moreover if $z \in X^0 \oplus X_1^-$, $\Lambda(z) < 0$.

Therefore by (Γ_1) again, $h_1(z) = z$ on $X^0 \oplus X_1^-$. Since $\varphi|_{X_1}$ satisfies (ψ_1) - (ψ_2) of Proposition 4.1, this result implies there is a $z_1 \in W_1 \cap M^-$ such that $h_1(z_1) \in X_1^+$ where $W_1 \equiv X^0 \oplus X_1^- \oplus \text{span}\{\varphi\}$.

We claim (z_1) is bounded in X . Let $z_1 = z_1^0 + z_1^- + z_1^+$. As was noted earlier, these components of z_1 are mutually orthogonal in L^2 . By Proposition 2.18 (iii), (z_1) is bounded in $L^2(S^1, \mathbb{R}^{2n})$. Since X^0 is finite dimensional and z_1^+ is a bounded multiple of φ , $(z_1^0 + z_1^+)$ is bounded in X . If (z_1^-) is unbounded, $\Lambda(z_1) = \|z_1^+\|^2 - \|z_1^-\|^2 \rightarrow -\infty$. Therefore for large i , $h(z_1) = z_1 = h_1(z_1) \in W_1 \cap M^- \cap X_1^+$, i.e. $z_1 = z_1^+$ for large i and (z_1) is bounded in X . It is clear that $(z_1^0 + z_1^+)$ possesses a convergent subsequence. By (Γ_2) ,

$$0 = P^- h_1(z_1) = e^{\varphi(z_1)} z_1^- + P^- P_1 Q(z_1)$$

or

$$(4.6) \quad z_1^- = -e^{-\varphi(z_1)} P^- P_1 Q(z_1).$$

Since Q is compact and $0 < \varphi(\cdot) < \gamma$, (4.6) shows (z_1^-) also has a convergent subsequence along which $z_1 \rightarrow z \in M^-$. Since h is continuous, $P_1 h(z_1) \rightarrow h(z) \in X^+$. Finally by (Γ_3) , $h(z) \in M^+$. Thus $h(M^-) \cap M^+ \neq \emptyset$ and the proof is complete.

Next we will give the

Proof of Theorem 1.2 for $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$: Let (\bar{H}_k) be a sequence of C^2 functions which are homogeneous of degree 2, satisfy (H_3) , and converge to \bar{H} in C^1 uniformly in a neighborhood of S^{2n-1} . The C^2 version of Theorem 1.2 implies there is a $z_k \in X$ which is a classical solution of

$$(4.7) \quad \dot{z}_k = \lambda_k \bar{H}_{kz}(z_k)$$

and $z_k(0) \in \bar{H}_k^{-1}(1)$. Equation (4.7) implies that

$$(4.8) \quad c_k \equiv A(z_k) = \frac{\lambda_k}{2} \int_0^{2\pi} z_k \cdot \bar{H}_{kz}(z_k) dt = 2\pi \lambda_k.$$

By Proposition 3.16, $c_k > 0$. Suppose that (c_k) is bounded away from 0 and ∞ . Then (4.8) shows the same is true for (λ_k) . Therefore (4.7) provides L^∞ bounds for \dot{z}_k and (4.7) and the Arzela-Ascoli Theorem imply a subsequence of (λ_k, z_k) converge in $\mathbb{R}^+ \times C^1$ to a solution (λ, ζ) of (2.4). Following the C^1 version of the proof of Lemma 2.3 then gives a solution of (HS) of the desired type. (Now β in (2.6) is merely continuous so (2.6) need not have a unique solution but any solution will suffice.)

It remains to get the bounds for c_k . By Proposition 3.16, there are constants \underline{a}_k , \bar{a}_k such that

$$(4.9) \quad \underline{a}_k < c_k < \bar{a}_k$$

where \underline{a}_k , \bar{a}_k are defined in (3.6), (3.7) with $M = M_k$. By (3.12), $\bar{a}_k < \pi m_k^{-1}$ where m_k is defined in (2.1). Since $\bar{H}_k \rightarrow \bar{H}$ uniformly on S^{2n-1} , $m_k \rightarrow m$ so for large k , $m_k > m/2$ and

$$(4.10) \quad c_k < \bar{\gamma}_k < 2\pi m^{-1}.$$

Thus (c_k) is bounded away from ∞ . To get a lower bound for c_k , recall that by (2.2),

$$\bar{H}_k(z) < M_k |z|^2$$

with M_k defined in (2.1) and $M_k \rightarrow M$ as $h \rightarrow \infty$. Hence $M_k < 2M$ for large k and if $z \in M_k$,

$$1 = \frac{1}{2\pi} \int_0^{2\pi} \bar{H}_k(z) dt < \frac{M}{\pi} \int_0^{2\pi} |z|^2 dt \equiv \Psi(z).$$

If $z \in \Psi^{-1}(1)$, there is an $a_k(z) > 1$ such that

$$\Psi_k(a_k(z)z) = \frac{1}{2\pi} \int_0^{2\pi} \bar{H}_k(a_k(z)z) dt = 1.$$

Therefore

$$(4.11) \quad \underline{a}_* = \inf_{z \in \Psi^{-1}(1) \cap \mathbb{R}^+} \Lambda(z) < \underline{a}_k$$

for large k . The argument of Proposition 3.8 shows $\underline{a}_* > 0$. Hence (4.9) and (4.11) show (c_k) is bounded from below and the proof is complete.

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20. ABSTRACT - cont'd.

(H₃) $H(p,q) = H(-p,q)$ for all $p,q \in \mathbb{R}^n$, then there is a $T > 0$ such that the Hamiltonian system

$$(HS) \quad \dot{z} = JH_z(z), \quad J = \begin{pmatrix} 0 & -id \\ id & 0 \end{pmatrix}$$

possesses a T periodic solution $(p(t), q(t)) \in H^{-1}(1)$ with p odd about 0 and $T/2$ and q even about 0 and $T/2$.

The proof involves a new existence mechanism which should be useful in other situations.

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